Divergent approximators

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1 Introduction

- We often mark uncertainty in our utterances with words like *maybe*, but when we mark uncertainty on numerals, unexpected things happen.
- Below we will see these unexpected effects described and explained using a possible world semantics analysis.
- This analysis will in turn inform our view on other scalar modifiers, like *approximately*.
- Additionally, it will help identify a complication for so-called *slack regulators* (e.g. *loosely speaking*, *exactly*), pointing to the unexplored importance of modality in differentiating approximators.
- Preview: Some approximators have modal components and behave differently from non-modal approximators (i.e they take into consideration contextual information).

2 Uncertain numerals

Puzzles

- You can use words like *maybe* to mark your uncertainty with respect to an item as in (1a), and as a result your interlocutor might entertain alternatives to this uncertain item, as sketched in (1b).

(1) a. A: Who won the race?
B: Maybe John.
b. {John, Ann, Pete}

- When the uncertain item is a numeral, there is a strong and somewhat surprising tendency for the set of alternatives to resemble approximation, as in (2).

(2) a. A: How many people competed?
B: Maybe twenty.
b. {18, 19, 20, 21, 22}

– cf. *Approximately twenty.*

- However, this does not occur for all uncertain numerals.

(3) a. A: Which player has the most fouls?
B: Maybe twenty.
b. {20, 6, 77, 15}

- Furthermore, when this approximation effect occurs, the range of alternatives depends on the numeral.

– Example: If you replace *twenty* in (2) with *twenty-seven*, the range tends to be smaller.

(4) a. A: How many people competed?
B: Maybe twenty-seven.
b. {26, 27, 28}

Summary of unexpected effects:

I. Why do uncertain numerals give rise to approximative readings, as in (2)?

II. Why do some uncertain numerals fail to give rise to approximative readings, as in (3)?

III. Why do some uncertain numerals give rise to more approximate readings than others, as in (2) vs. (4)?

Explanation

Preview:

– Scalars represent ranges of values.
– This range information is used in determining alternatives to uncertain numerals.

- These phenomena can be given a formal explanation using Krifka (2009)'s conception of numerals, along with a possible world semantics as described in Kratzer (1991).

– Following Kratzer (1981/1991)
Modal base:

> "...determines for every world the set of worlds which are accessible from it." (Kratzer 1981:47)

Ordering source:

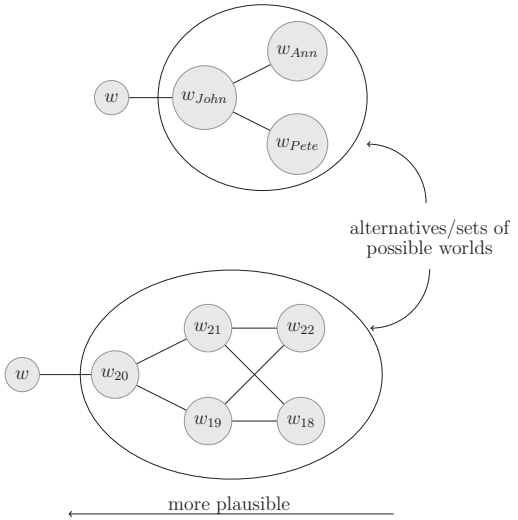
A set of propositions A induces a partial ordering $\leq_A$ on W in the following way (Lewis 1981):

For all w, w' ∈ W, for any A $\subseteq \mathfrak{B}(W)$:

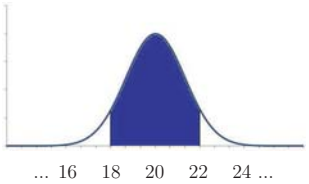
$w \leq_A w'$ iff $\{p : p \in A \text{ and } w' \in p\} \subseteq \{p : p \in A \text{ and } w \in p\}$

(Kratzer 1991:644)

- Consider alternatives to be sets of possible worlds (i.e. worlds consistent with the epistemic modal base). These sets of worlds will be ordered in terms of their plausibility by an ordering source.



- Following Krifka (2009) we can assume that numerals represent a range which can be characterized as the values which fall within one standard deviation (σ) of the uttered numeral (μ) on a normal distribution over the number line.
  - Example: The sentence *This book cost $20*, used in a relatively imprecise context. Assume $\sigma = 2$, *twenty* then represents values in the range $[18 - 22]$



  - The normal distribution represents probability of each value being true. Beyond one standard deviation (i.e. beyond the shaded area), probability is too low.
  - Note that I assume, for simplicity, strict cut-offs at $\pm\sigma$. It seems, however, that the border should be fuzzy, and this fuzziness could result from difficulty determining a precise σ.[1]

- Phrased in terms of propositions, this gives p_σ (5), which picks out worlds where the value intended by the speaker (y) falls within one standard deviation (σ) of the uttered numeral (μ), and a family of functions p_x (6), which pick out worlds where the intended value (y) falls within $\sigma - x$ of that number (μ) for $0 < x < \sigma$.

  Let y assign to any world the numeric value intended by the speaker in that world (representing public uncertainty about what value the speaker intends).

  (5) $p_\sigma = \lambda w.y(w) \in \{[\![\mu - \sigma]\!], ..., [\![\mu + \sigma]\!]\}$
  ($y(w)$ = value intended by speaker in w, μ = value uttered)

  (6) $p_x = \lambda w.y(w) \in \{[\![\mu - x]\!], ..., [\![\mu + x]\!]\}, 0 < x < \sigma$

  - Example: said *20*, by context assume $\sigma = 2$
    $p_\sigma = \lambda w.y(w) \in \{[\![20 - 2]\!], ..., [\![20 + 2]\!]\}$ (y = actual value, $\mu = 20$)
    i.e. picks out set of worlds where the value y intended by the speaker in that world is between 18 and 22
    $p_x = \lambda w.y(w) \in \{[\![20 - x]\!], ..., [\![20 + x]\!]\}, 0 < x < 2$

Quick recap:
- alternatives as sets of possible worlds
- numerals represent ranges ($\pm\sigma$), expressed through propositions (p_σ, p_x)

- I will treat *maybe* as involving a sort of modal possibility operator.

[1] Cf. Possibility of halos as fuzzy sets in Lasersohn (1999).

- For uncertain numerals (e.g. *maybe twenty*), the modal base will contain the sets of worlds consistent with p_σ and the ordering source will contain the worlds consistent with the propositions in p_x for $0 < x < \sigma$.
- We now have an explanation for the approximation that arises from uncertain numerals:
  - Only worlds where values close to the uttered numeral are true will be accessible, so only these values will be plausible alternatives.
- We also have an explanation for why approximation does not always occur with uncertain numerals:
  - This effect only happens with *scalar* numerals, like in (2), not with numerals acting in a non-scalar labeling capacity (3) which do not represent normal distributions.
  - Note that non-scalars, as in (1), can still be seen as approximate in a non-numerical sense, which is consistent with this explanation (discussed below).
- And finally if we consider Krifka's pragmatic preference for simple expressions[2], we have an explanation for why the range of alternatives depends on the numeral (why *maybe twenty* in (2) leads to a wider range of alternatives than *maybe twenty-seven* in (4)):
  - Krifka — preference for simple expressions leads more complex numerals like *twenty-seven* to represent smaller ranges (i.e. induce smaller σs) than simpler numerals like *twenty*.
  - Since *twenty-seven* has a smaller σ, its p_σ allows a smaller range of possible worlds, leading to its narrower interpretation as an uncertain numeral (see Krifka 2009 for details).

Summary of explanations

I. Uncertain numerals give rise to approximative readings because they involve p_σ/p_x in their modal base/ordering source, so possible worlds are those in which the numeral is close to the uncertain numeral.

II. Some uncertain numerals fail to give rise to approximative readings because they are not scalar and therefore are not associated with p_σ and p_x

III. Some uncertain numerals give rise to more approximate readings than others because they are associated with larger σs, so p_σ allows a wider range of possible worlds.

[2]Here we can assume that shorter = simpler/less complex, but the story is more complicated than that (see Krifka 2009).

Beyond numerals

- Numerals are not unique in expressing ranges, so this effect should not be unique to numerals.
- This analysis for the approximative effect of uncertain numeral extends naturally to other scalars, suggesting that all scalars behave alike in representing a range characterized by a normal distribution.
- Example: color

(7) a. A: You say you got a good look at John's car. What color is it?
B: Maybe blue.
b. {}

(8) a. A: You say you got a good look at John's car. What color is it?
B: Maybe cyan.[3]
b. {}

  - Note: colors are also subject to another kind of approximation, termed *epistemic* by Sauerland and Stateva (2007), which concerns uncertainty as to the precise meaning of the word in question (here *blue*/*cyan*).
- In fact, this approximation effect can be seen with any item that is used scalarly.
- Example: beef stroganoff
  - Consider a scalar interpretation of beef stroganoff (as in *Well, it was only approximately beef stroganoff*).
  - Using this scalar interpretation, consider *What Mary cooked was maybe beef stroganoff*.
  - This gives the reading that what Mary cooked was somewhere near the ideal of beef stroganoff, or approximately beef stroganoff.
  - See Sauerland and Stateva (2007) for a different take on this kind of construction.

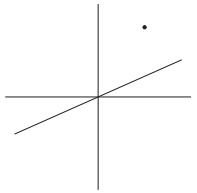
(9) Judgements from Sauerland and Stateva (2007)
a. What John cooked was definately/maybe beef stroganoff.
b. # What John cooked is exactly/approximately beef stroganoff.

They consider (9b) bad because scalar approximators (*exactly*/*approximately*) can only combine with scalar items. Here, I suggest that *beef stroganoff* **can** be scalar, and when it is, ***maybe beef stroganoff* gives rise to a similar type of scalar approximation** (like *maybe*/*approximately twenty*).

Note that (9a) gives rise to a different kind of approximation, namely epistemic approximation, when *beef stroganoff* is epistemically (not scalarly) vague.

[3]In case you are not familiar with it, cyan is this color: 

- Back to *maybe John* in (1) — could {John, Ann, Pete} be thought of as approximation too?
  - Think of *John* as representing a point on some scale(s)
  - Alternatives are like John in certain relevant respects
  - Example:



x = speed
y = susceptibility to performance anxiety
z = predisposition to race

Summary:
- The same phenomena we saw with uncertain numerals happen with other scalars.
- The same explanation applies.

3 Other approximators

- This analysis of uncertain numerals can inform the way we think of other means of approximation.
- Compare *approximately*
  - Gives rise to approximate readings (e.g. *approximately twenty people*)
  - Does so by expressing that something falls within a range (e.g. that the number of people falls within some range around twenty), perhaps with a denotation like (10)[4]
  - Does not give rise to alternatives like *maybe* (difference may be subtle at this point, should be clearer as we work through *approximately*)

Again, μ =uttered numeral.

[4]For *approximately*'s counterpart, perhaps: $[\![\textbf{exactly}]\!] = [\lambda n.\lambda y.\exists z \in \{[\![\mu_n - \sigma_n]\!], ..., [\![\mu_n + \sigma_n]\!]\}|\#y = z$, defined if $\sigma_n < \sigma_{c,n}]$

(10) $[\![\textbf{approximately}]\!] = [\lambda n.\lambda y.\exists z \in \{[\![\mu_n - \sigma_n]\!], ..., [\![\mu_n + \sigma_n]\!]\}|\#y = z]$
(takes a scalar n and some y and returns true if the location of y is within σ of n on the relevant scale)

- Example: *approximately twenty people*
  $[\![\textbf{approximately twenty people}]\!]$
  $= [\lambda n.\lambda y.\exists z \in \{[\![\mu_n - \sigma_n]\!], ..., [\![\mu_n + \sigma_n]\!]\}|\#y = z]([\![\textbf{twenty}]\!])([\![\textbf{people}]\!])$
  $= [\exists z \in \{[\![\mu_{20} - \sigma_{20}]\!], ..., [\![\mu_{20} + \sigma_{20}]\!]\}|\#people = z]$
  Assume $\mu_{20} = 20, \sigma_{20} = 2$
  $= [\exists z \in \{18, ..., 22\}|\#people = z]$
  (there is some number in the range $[18 - 22]$ which is equal to the number of people, i.e. the acutal number of people is in the range $[18 - 22]$)

- This denotation incorporates σ, allowing *approximately* to show the same range effects as *maybe* (cf. *approximately twenty* and *approximately twenty-seven*)
  - As we saw in Krifka, σ is generally smaller for more complex numerals like *twenty-seven*, and since σ determines the range for *approximately*, *approximately twenty-seven* has a narrower range than the simpler *approximately twenty*.
- But this denotation captures an important difference, shown in (11).

(11) a. It's Susan's birthday today, and she's maybe thirty.
b. # It's Susan's birthday today, and she's approximately thirty.

- Here *approximately* is unable to accommodate the fact that it is Susan's birthday (i.e. that ages like 31 and three months are impossible).[5]
- With *maybe*, on the other hand, this information can easily be accommodated in the modal base, excluding incompatible ages.
- This is reflected in the denotation above in (10), where z is drawn from a continuous range.
  - Note that *approximately* is still technically consistent with it being Susan's birthday, but it suggests that intermediate values are possible. This results in strangeness, requiring a certain amount of work/inference on hearer's behalf.

[5]Note that *approximately* is acceptable in a very precise context (e.g. *Actually, she's 29 years 14 hours and 22 minutes*), but this is not the reading that I am considering.

- So, through associating scalars with the kind of information described by Krifka, the similarities between *maybe* and *approximately*, as well as their differences, can be captured.

approximately	*maybe*
non-modal	modal
does not accommodate contextual information	accommodates contextual information
uses σ for range	uses σ for modal base

  - The account in Sauerland and Stateva (2007) may be able to capture this as well: *approximately* relies on scale (it adjust scale granularity, non-modally), but *maybe* is quite different (combine with predicates without known precise meanings, quantifies over possible extensions).

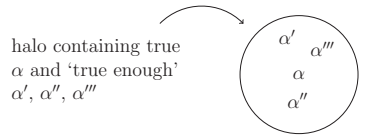
- But what about (2)? Note that *approximately twenty people* is less offensive than (11b).
  - Solution: atomicity
  - People are considered atomic, and so only integer-increments of people are considered. Years, on the other hand, are readily divisible.

- cf. *approximately John*
  - *maybe twenty* $\approx$ *approximately twenty*, but *maybe John* $\not\approx$ *approximately John*
  - i.e. the alternatives you get for *maybe John* do not tend to be the same as the items that fall within the denotation of *approximately John*
  - *Maybe John* as more macro, perhaps due to contextual information accommodation: you are presumably searching for actual people, not purely hypothetical John-like people, so the range (σ) needs to be wider if it is to include any alternatives.

4 Halos

- The analysis presented above is reminiscent of Lasersohn (1999)'s pragmatic halos.
- Some element α is surrounded by a halo of elements which differ from α in pragmatically ignorable ways.[6]

[6]Lasersohn writes: "Given an expression α denoting some object x, I like to think of the set the context associates with x as arrayed around x in a sort of circular cluster, so I will call this set,together with its ordering relation, the PRAGMATIC HALO of x, or, extending the terminology, as the pragmatic halo of α", (Lasersohn 1999:527) and "$H_C(\alpha)$ is understood to be a set of objects which differ from $[\![\alpha]\!]^{M,C}$only in ways which are pragmatically ignorable in C; $\leq_{\alpha,C}$ is an ordering of $H_C(\alpha)$ according to similarity to $[\![\alpha]\!]^{M,C}$", (Lasersohn 1999:548).



- It would seem that the propositions in the modal base and ordering source above are the same as the information structuring these pragmatic halos (i.e. the information used to determine what is pragmatically ignorable and how to order based on similarity).[7]
- However, one difference soon becomes apparent, which is seen most clearly through slack regulators.
- Pragmatic halos can be manipulated using slack regulators like hedges (e.g. *roughly*, *loosely speaking*), which more or less expand $[\![\alpha]\!]$ to include its halo[8].
  - Example: While $[\![$**twenty**$]\!]$ is only true for 20 exactly, $[\![$**roughly twenty**$]\!]$ is true for values that differ from twenty in pragmatically ignorable ways.
- Now, to see how the information used in the possible worlds account differs from one using pragmatic halos, compare the use of *maybe* with the hedge *roughly* in (12).

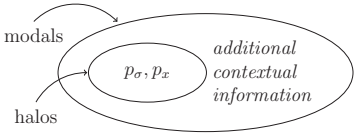
(12) a. It's Susan's birthday today, and she's maybe thirty.
  b. # It's Susan's birthday today, and she's roughly thirty.

- Again, *maybe* can readily accommodate the fact that it is Susan's birthday, **but with *roughly*, this does not have the same effect on the halo**, leading to infelicity.[9]
- This is not *roughly*-specific.
  - Even round numbers (e.g. *twenty* when it represents $[18-22]$) do not accommodate this kind of outside information.
- **So, while there is overlap in the information structuring pragmatic halos and the information structuring possible worlds, the overlap is not complete.**
- What is the difference, exactly?
  - Halos deal with precision (p_x, p_σ) only
  - Modals accommodate precision as well as additional contextual information

[7]Modal base: "close enough not to obscure pragmatically relevant details or distinctions"; Ordering source: closeness according to some dimension

[8]$[\![$**loosely speaking** $\Phi]\!]^{M,C} = \bigcup H_C(\Phi) - [\![\Phi]\!]^{M,C}$

[9]Note that *roughly* (like *approximately*) is acceptable in a very precise context.



- Note: modal approximators involve uncertainty
- Now that this distinction has been noted, we may expect find items like *maybe* which have been mis-classified as slack regulators.
- Example: Siegel (2002)'s *like*
  - In her semantic denotation, *like* α denotes a variable corresponding either to α or an element within α's halo. As can be seen in (13), however, *like* can accommodate outside information, just like *maybe*.

    (13) It's Susan's birthday today, and she's, like, thirty.
  - This cannot be explained by halos and suggests that there is some modal semantic component to *like* such that outside information can be accommodated in its modal base, explaining the felicity of (13).
- Short list of approximators divided by camp:

  Modal: *maybe*, *like*

  non-modal: *approximately*, *roughly*

Summary:
- Halos seem to be a similar way to determine alternatives/approximation.
- But halos involve precision only, like approximation (non-modal).

5 Summary

- By examining constructions like *maybe twenty* it can be shown how information associated with numerals can be incorporated into a possible worlds semantics to describe their behavior and their divergence from constructions like *approximately twenty*.
  - Scalars represent ranges, with closer values being more probable.

  - In modal contexts (e.g. *maybe* x), this information is incorporated into the modal base and ordering source such that plausible alternatives are those scalarly close, resembling approximation.
- It can also be seen that, while this same information may be used in pragmatic halos, use of contextual information sets these types of approximation apart and suggests that certain hedges contain modal components.
  - Modal approximators accommodate contextual information, while non-modals cannot.

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A More support for this modal contrast

From Geurts and Nouwen (2007):

- Superlative modifiers (e.g. *at least/most*) are modal
- Comparative modifiers (e.g. *more/less than*) are not

- Turns out that *maybe* patterns like superlatives (modal) and *approximately* patterns like comparatives (non-modal) on at least two fronts discussed in Geurts and Nouwen (2007):

- Specificity

(14) a. I will invite {at most two people/maybe} two people, namely Jack and Jill.

b. ? I will invite {fewer than/approximately} three people, namely Jack and Jill

- Distributional restrictions

(15) a. Betty had three martinis {at most/maybe/*fewer than/?approximately}.

b. {At least/Maybe/*More than/*Approximately}, Betty had three martinis.