Expressing numerical uncertainty*

Erin Zaroukian

Draft: February 26, 2010

1 Introduction

There are many ways to express a number without fully committing to it. For example, consider the sentences in (1) which express only *loosely* the number of books John read.

(1) a. John read approximately twenty books.
 b. John read about twenty books.
 c. John read twenty books, more or less.
 d. John read twenty-ish books.
 e. John read maybe twenty books.
 f. John read something like twenty books.

Russian, among the other East Slavic languages, has a somewhat exotic way to express a number loosely called Approximative Inversion (AI), as demonstrated in (2). (2a) provides the basic word order, and in (2b) the noun and the numeral have inverted, yielding and approximative reading of the numeral.

(2) a. Ivan pročital dvadcat' knig.
 Ivan read twenty books
 'Ivan read twenty books.'
 b. Ivan pročital knig dvadcat'.
 Ivan read books twenty
 'Ivan read approximately[1] twenty books.'

Linguists investigating AI have focused on its syntax and have generally considered it an instance of head movement. Here, I will focus attention rather on the semantics of AI and provide an analysis of AI as a marker of speaker uncertainty which incorporates information associated with the expressed numeral. I will then review the head movement analysis of AI and show it to be generally incompatible with the semantics of AI. Finally, I will present a more semantically-compatible syntactic analysis of AI which treats the numeral as a post-nominal modifier.

*This paper is based on a presentation with the same name delivered 7 January 2010 at the 84th annual meeting of the LSA. Questions and comments are welcome (zaroukian@cogsci.jhu.edu).

[1]Following the literature, I will be using *approximately* in AI glosses, though as will be seen later this is not quite accurate.

2 Semantics of AI

In this semantic analysis, I will be interested in explaining both why AI is semantically felicitous in certain contexts but not others, as well as what AI means when it is felicitous. To begin, we can first step back and think about the sentences in (1). While these sentences each provide a loose expression of the number of books John read, they do not mean the same thing. For example, compare "approximately twenty books" in (1a) with "maybe twenty books" in (1e). Here, "approximately" seems to indicate that the numeral is being used imprecisely and that the speaker's claim is that the number of books John read falls within some range around twenty.[2] "Maybe", on the other hand, seems to indicate that the speaker is uncertain how many books John read.[3] Given that it had been termed *Approximative* Inversion, you may expect AI to pattern like "approximately" and to express the same type of imprecision. Native speakers, however, have the general intuition that the use of AI expresses uncertainty on the part of the speaker, suggesting that AI actually patterns with "maybe" in marking speaker uncertainty. As will be seen below, however, AI does not act straightforwardly as an approximator *or* as a marker of speaker uncertainty.

Consider first the scenario in (3), borrowed from Pereltsvaig (2006).

(3) Birthday example: (Pereltsvaig 2006:284)
Masha is going to a colleague's birthday party and is asked how old that colleague is. Since she doesn't know him very well, she is guessing his age from his looks, etc. In this situation, Masha's reply can use the approximative inversion in [(3a)], but not any other approximative strategy, such as using *priblizitel'no* 'approximately' or an interval:

a. let tridcat'
years thirty

b. # priblizitel'no tridcat' (let)
approximately thirty years

c. # 30-35 let
30-35 years
'approximately thirty years'

Here we see that AI is appropriate in a context where the speaker is uncertain about the number she is expressing. We also see that AI patterns *against* the approximators in (3b) and (3c). In this scenario, then, AI appears to mark uncertainty, not approximation.

Now consider the scenario in (4).

(4) Zodiac example:
You're talking to an acquaintance, and she tells you her brother was born in the year of the ox, which for present purposes means he's 11, 23, 35, 47, 59, 71, or 83 years old. This acquaintance is in her thirties, so your best guess would be that her brother is 35 (as opposed to 11, 23, etc.).

a. # let tridcat' pjat'
years thirty five
'approximately thirty-five years'

[2] This may involve a denotation along the lines of ⟦**approximately**⟧=$\lambda n.\lambda P.\exists y \in \{n-x_c, ..., n+x_c\}$ s.t. $P(y)$, which says that there is some number falling within a contextually-defined range of the expressed numeral that makes $P(y)$ true.

[3] This may involve a denotation along the lines of ⟦**maybe**⟧=λn s.t. $uncertain(s_c)(n).n$, which presupposes that the speaker is uncertain with respect to the numeral n.

Here, as in (3), the speaker is unsure about the number she is expressing, but *unlike* (3), AI is not felicitous. So, while (3) suggests that AI expresses uncertainty instead of approximation, (4) suggests otherwise.

In what follows, I will maintain that AI marks the speakers uncertainty with respect to the numeral expressed. The felicity contrast between (3) and (4), as well as the fact that AI often results in approximative readings of the numeral, will be explained in terms of information associated with the expressed numeral.

2.1 Deriving approximation from uncertainty

I propose that in AI the expressed numeral contributes closeness information which the hearer uses in determining alternatives, leading to a set of alternatives that may look like approximation, and this closeness information can potentially be incompatible with other information available to the speaker. To see how this will work, let us first step back and consider the effect of uncertainty in general.

When a speaker utters some X and marks their uncertainty with respect to it, a hearer may wish to entertain alternatives to X, and to come up with these alternatives, the hearer will use any relevant available information. For example, if a speaker is not sure what exactly John read, he might say that is was "maybe a newspaper", thereby uttering "newspaper", but indicating his uncertainty with respect to it. Since the speaker has expressed this uncertainty, the hearer may wish to entertain alternatives to "newspaper" and may come up with a set of relatively-high-probability alternatives, like {newspaper, magazine, book}, by using world knowledge. And if other relevant information is available, the hearer should use this as well. For example, if it was known that John is an avid reader of receipts (which would otherwise be a low-probability alternative to "newspaper"), the set might instead be {newspaper, receipt}.

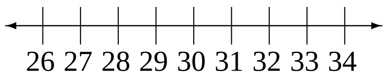
We are interested in the case where the X marked as uncertain is a scalar numeral. Scalar numerals are defined with respect to a scale, unlike labeling-type numbers (e.g. bus numbers (Wiese 2003)), and this scale provides information about what is similar to that numeral. To see this, consider 20 on the number line in (5). This scale tells you that the numbers that are least different from 20 are those which are closest to it. For example, 21 is quite like 20, even more so than 22.

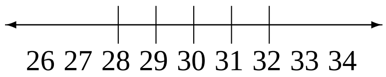
(5) 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29

If this similarity information associated with the numeral is used in computing alternatives, you would expect more similar numbers to be more likely alternatives. So, we can see how this can lead to what looks like approximation, i.e. a set of alternatives like {X-2, X-1, X, X+1, X+2}. In AI, I claim that this closeness information not only can be, but *is* used in computing alternatives. This can be explained in neo-gricean terms: the speaker wouldn't have gone to the trouble of using a scalar numeral carrying this closeness information unless that information were relevant, e.g. for computing alternatives. Therefore, the hearer faces pragmatic pressure to use this information in computing alternatives.

Let us now see how this analysis proceeds in (3), where AI is felicitous. Here, the speaker uses AI, marking her uncertainty with respect to the numeral. The hearer may then entertain alternatives to that numeral. Since the speaker used the scalar 30, the hearer faces pragmatic pressure to use the numeral's closeness information in computing alternatives and only entertain ages close to 30 (e.g. (6a)). The hearer also knows that it is this colleague's birthday and will therefore only entertain alternatives like 29 or 32, not 31;2 or other such ages (e.g. (6b)). So, by using both the information contributed by the numeral and the knowledge that it is this colleague's birthday, the hearer can come up with a set of alternatives appropriate to this context (e.g. (6c)).

(6) a. 
26 27 28 29 30 31 32 33 34

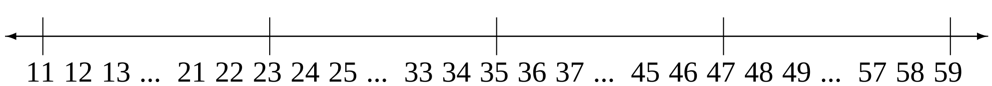
b. 

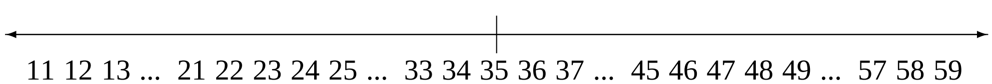
c. 

Note that we now have an explanation for the infelicity of approximators (3b) and (3c). The problem with these utterances is that they are too continuous and suggest that the colleague might in fact be 31;2, which is incompatible with the context. It appears that approximators such as in (3b) and (3c) are less receptive to outside information and the fact that it is the this colleague's birthday does not rule out intermediate ages, leading to infelicity. This may be because approximators do not encourage the hearer to entertain alternatives like uncertainty markers do and therefore lack this opportunity to make use of relevant information in the computation of alternatives. Rather, approximators express that X falls within some range. Note that the same appears to be true in English (compare "maybe 30" and #"approximately 30" for the context in (3)).

Now consider (4), where AI is not felicitous. A speaker using AI in this scenario would mark their uncertainty with respect to the numeral as in (3), and again the hearer would be encouraged to entertain alternatives. Since the speaker used the scalar 35, the hearer would face pragmatic pressure to use the numeral's closeness information in computing alternatives and only entertain alternatives close to 35. Here, another type of closeness information becomes important. We have discussed how the numeral is associated with information about what is similar to itself, namely what is close to itself, but the numeral is also associated with information about what is sufficiently similar to itself, namely what it *close enough* to itself. To get a sense for this, consider round numbers. For example, in the right context you can use the numeral 35 to express a quantity you measured as 33, but you are unlikely to find a context where you could use 35 to express 23: 23 just isn't close enough. Now, given that numerals are also associated with information about what is close enough, hearers should use this information as well when computing alternatives. In (4), then, the hearer will entertain alternatives which are close to 35, but only those which are *close enough* to 35 (e.g. (7a)). However, the hearer also knows that this brother is 11, 23, 35, 47, etc. (e.g. (7b)), and a problem arises when these pieces of information are brought together (e.g. (7c)). Specifically, none of the alternatives provided by the knowledge that this brother was born in the year of the ox is *close enough*, i.e. even the closest alternatives, 23 and 47, are inconsistent with the information provided by the numeral 35. The only alternative left to the hearer is 35 itself, which is inconsistent with the speaker's uncertainty (if this brother must be 35, how could the speaker be uncertain?), leading to the infelicity of AI in (4).

(7) a. 

b. 

c. 

We have now seen how an analysis of AI as a marker of speaker uncertainty can account for the data. Closeness information contributed by the numeral and utilized by they hearer can lead to sets of alternatives which look like approximation, and the utilization of other relevant information can cause sets of alternatives to less resemble approximation (compare (3a) with (3b) and (3c)). We have also seen how the infelicity of AI in (4) can be explained by the incompatablilty of information used to compute alternatives. Now it remains to be seen how such a semantic analysis could be realized syntactically.

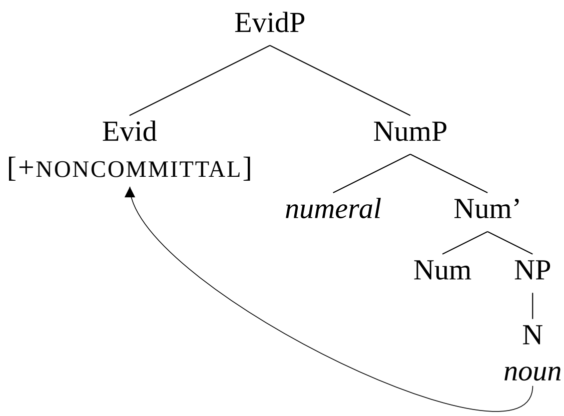
3 Syntax of AI

AI has generally been viewed as head movement of the noun to check some approximation-related feature, and while this analysis has certain advantages, we will see that it is ultimately problematic semantically.

3.1 Head Movement analysis

At first glance, head movement provides a rather tidy account of AI. Consider the structure in (8), proposed by Pereltsvaig (2006). Here the noun moves to the left of the numeral, resulting in the correct word order, and this movement is motivated by feature checking. The relevant feature, [+NONCOMMITTAL], is one which marks the speaker's public uncertainty. Thus, head movement seems to provide a rather parsimonious explanation for the meaning and form of AI.

(8) (based on Pereltsvaig 2006)



There is further evidence pointing toward a head movement analysis. For one, it seems that AI cannot move anything larger than the noun. In (9), you can see this in the form of PP-complement stranding. (9a) depicts the basic word order. When AI is applied, the resulting word order is that in (9b), with the PP-complement stranded, not that in (9c), where the entire phrase has moved. This is predicted by a head movement analysis, since it is only the head, without its complement, which moves.

(9) PP stranding (Pereltsvaig 2006:278)

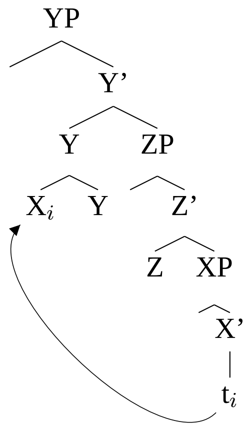
a. desjat' [pobed [$_{PP}$ nad vragom]] (non-inverted)
ten victories over enemy$_{INST}$
'ten victories over the enemy'

b. pobed desjat' [[$_{PP}$ nad vragom]] (inverted)
victories ten over enemy$_{INST}$
'approximately ten victories over the enemy'

c. * [pobed [$_{PP}$ nad vragom]] desjat' (*inverted)
victories over enemy$_{INST}$ ten

Another piece of evidence involves the head movement constraint, which states that a head may not skip an intervening head, as shown in (10).

(10) A HMC violation:



Now note the data in (11). When an adjective is involved, AI is impossible. The noun cannot move past A, since it is an intervening head, and the phrase including the adjective cannot move, since AI is *head* movement.

(11) Light adjectives[4] (adapted from Pereltsvaig 2006:279)

a. desjat' dovol'nyx lingvistov
ten satisfied linguists
'ten satisfied linguists'

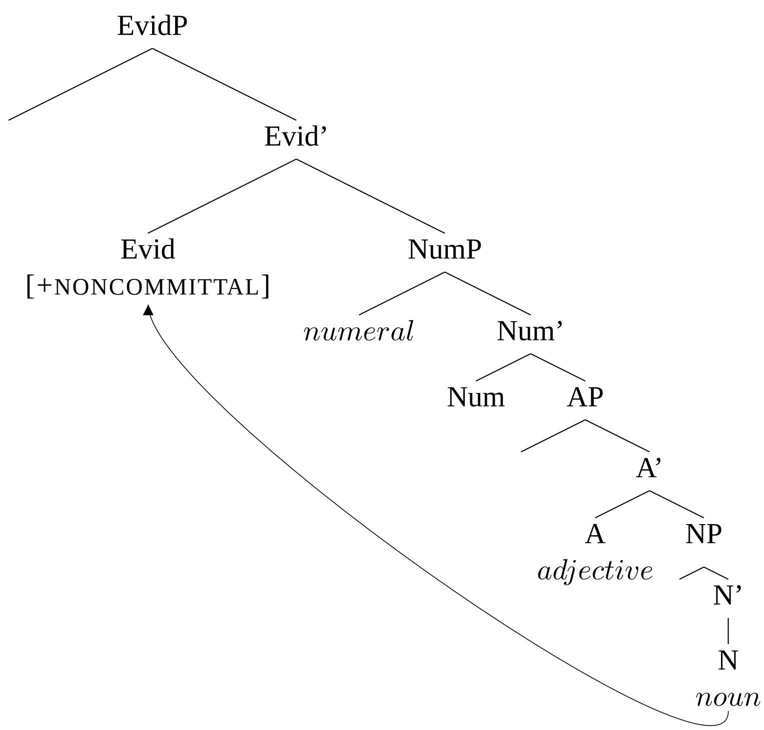
b. (*dovol'nyx) lingvistov (*dovol'nyx) desjat' (*dovol'nyx)
(satisfied) linguists (satisfied) ten (satisfied)
'approximately ten satisfied linguists'

If the adjective occupies an intervening head, this pattern is expected, since the noun would be required to skip an intervening head, as shown in (12), violating the head movement constraint.

[4] Heavy adjectives, on the other hand, are possible in AI, and it has been claimed that this is because they occupy a specifier position, not a head.

(1) lingvistov desjat' [$_{AP}$ dovol'nyx svoimi vystuplenijami] (Pereltsvaig 2006:279)
linguists ten satisfied self's$_{INST}$ talks$_{INST}$
'approximately ten linguists satisfied with their own talks'

(12)

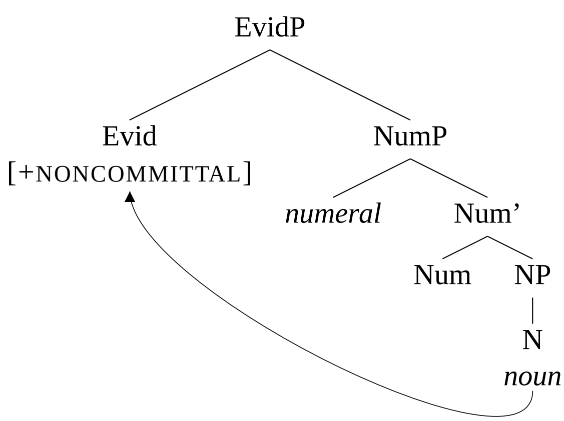


Ultimately, however, AI is not as clean cut as a head movement analysis suggests. First, it is not clear that adjectives should be considered intervening heads (cf. analyses which place the AP in specifier position such that A does not intervene). Additionally, consider (13). Here, it appears that the noun has skipped an intervening head containing the preposition, and yet the utterance is grammatical.

(13) knig za pjat' (Billings and Yadroff 1996:46)
books$_{\text{GEN.PL}}$ for five
‘for approximately five books’

The most formidable problem facing a head movement analysis, however, is semantic. Consider again the structure in (8), repeated below in (14). Here, the noun head-moves to check a feature to the left of the numeral marking the speaker’s uncertainty. This is problematic semantically because the feature is being checked by the *noun*.

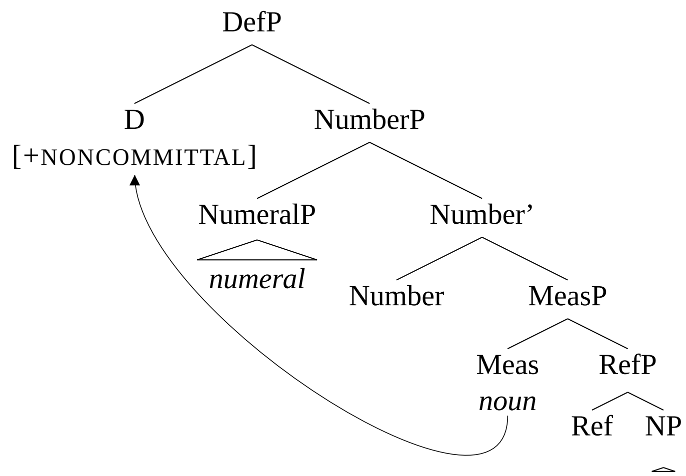
(14) (based on Pereltsvaig 2006)



Consider the interpretation that would be expected from (14). A compositional account predicts ⟦**N [+NONCOMMITTAL]**⟧ = noncommittal(N), which, for a sentence like (2b), should mean that Ivan read 20 things which were maybe books. This, however, is not what the sentence means. Rather, the sentence means that Ivan read some number of books which was maybe 20, i.e. the uncertainty is with respect to the numeral, not the noun.

Yadroff and Billings (1998) recognized this problem and, instead of head-moving N, they head-moved Meas, as shown in (15).

(15) (based on Yadroff and Billings 1998)[5]



Meas is the head of a functional projection where [COUNT] is checked, and it is where units or measure words appear. For example, in (16) *kilogrammov* 'kilos' would be in Meas, as would *štuki* 'items' in (17).

(16) On pročital pjat' kilogrammov knig.
he read five kilos books
'He read five kilos of books.'

(17) My kupili štuki tri krasivyx plat'ev. (Yadroff and Billings 1998:335)
we bought item three pretty dresses
'We bought approximately three pretty dresses'

However, when we consider the interpretation expected from (15), a compositional account predicts ⟦**Meas [+NONCOMMITTAL]**⟧ = noncommittal(Meas), which, for a sentence like (2b), should mean that Ivan read 20 books, but you're unsure about the unit. This, again, is not what the sentence means.[6]

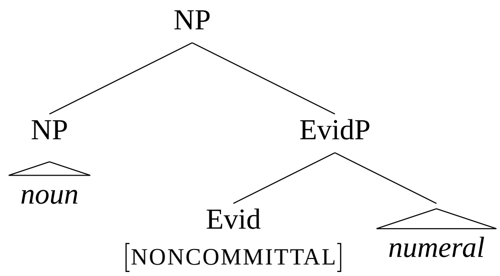
[5]Yadroff and Billings used the feature [-DEF] instead of [+NONCOMMITTAL]. They claim that AI is in complementary distribution with [+DEF] expressions, though they do not seem to provide (clear) supporting data, and they show how AI behaves like [-DEF] expressions in terms of scope effects (AI expressions do not seem to have their own quantificational force). They view AI, then, as head movement to check the formal feature [-DEF] on Meas, resulting in an indefinite measure reading. This, however, misses the epistemic component of AI. Additionally, it seems to conflate different notions of indefinite, namely, the formal or perhaps non-unique sense of indefinite and the ill-defined sense and it suggests that all [-DEF] expressions should involve inversion, which is not the case. It is not clear how checking this formal feature should result in an ill-defined reading. Since [-DEF] is problematic, and in the interest of consistency, I use the feature [+NONCOMMITTAL] in my discussion (though it may be less obvious why this feature should be housed in D).

[6]Yadroff and Billings seem to assume that checking a feature on Meas results in that feature being interpreted with respect to the numeral, not the unit of measure itself. This, as they point out, is similar to Krifka (1995), where the contents of M (i.e. Meas)

3.2 A new analysis

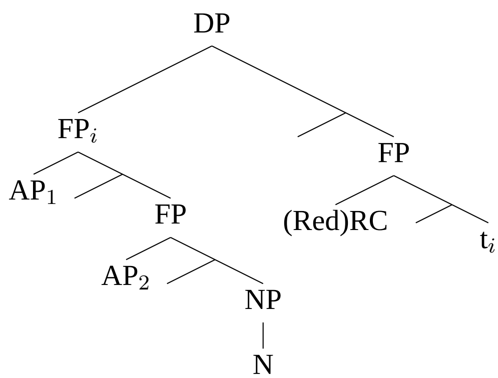
Given these problems associated with the head movement analysis, a different type of syntactic structure seems in order. The semantics of AI suggests a structure like that in (18), where instead of relying on approximative-feature checking to move the noun to a pre-numeral position, the numeral in AI is generated in a post-nominal position. This achieves the correct word order while associating the numeral, not the noun, with [NONCOMMITTAL].

(18)



This type of structure resembles Cinque (2005)'s analysis of post-nominal modification, which involves a post-nominal (reduced) relative clause as in (19).

(19)

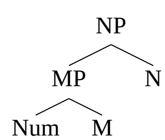


This structure also makes AI parallel to similar Russian post-nominal modification constructions, shown in (20) with a structure provided in (21).[7]

---

operate as a function which, when applied to an object, yields the number of relevant units that object consists of. To achieve this, however, Krifka uses the following syntactic rules: MP → Num M and NP → MP N, i.e. (1).

(1)



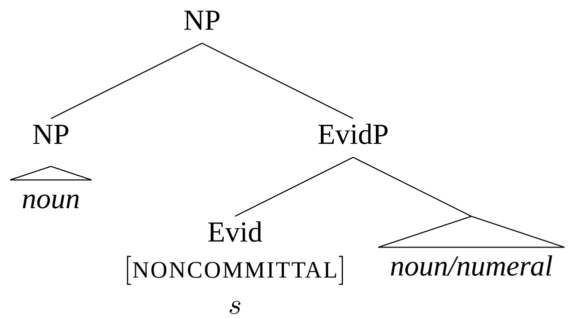
In Billing and Yadroff's syntax, however, it is less clear why the numeral and Meas should compose to give such an interpretation. Additionally, MP (or MeasP) cannot be interpreted simply as a number, it must be a number *of something*, i.e. the information that was/will be contributed by the numeral cannot be ignored if the correct semantics are to be achieved. However, if this information is included when uncertainty is marked, we once again seem to be marking uncertainty with respect to the noun. For example, checking [+NONCOMMITTAL] on Meas (*štuki*, 'items') in (17) you would be marking uncertainty not just with respect to three items, but with respect to dresses as a unit of measure as well. By Krifka's semantics, for example, $[\![\textbf{kilogrammov}]\!] = \lambda n \lambda y \lambda i \lambda x[\textbf{RT}_i(x,y)$ & $\textbf{kilo}_i = n]$ (i.e. x is a specimen/subspecies of type y and the number of kilograms of x is n), so marking uncertainty on Meas (or M) will mark uncertainty on more than just the numeral.

[7] For (20b) to work with this semantic analysis, it seem that the second noun must be thought of in a scalar-size sense.

(20) a. osetrof s sorok (Billings 1995:12)
sturgeons-GEN.PL S forty-ACC
'about forty sturgeons (archaic)'

b. mal'čik s pal'čik
boy S thumb-ACC
'boy the size of a thumb, Tom Thumb'

(21)



While the structure proposed in (18) is much more semantically coherent, it does not immediately solve the syntactic puzzles presented above. However, I would like to briefly suggest possible solutions. First, regarding the PP-complement stranding and impossibility of light adjectives presented in (9) and (11a), the structure in (18) might involve head movement of the noun, it just would not be [NONCOMMITTAL] that motivates it.[8] Alternatively (or perhaps in conjunction), there may be prosodic constraints on the elements involved in AI, as suggested by Billings (1995).[9]

4 Summary

Here we have seen how the semantic distribution and interpretation of AI can be accounted for by analyzing it as a marker of speaker uncertainty: By marking their uncertainty with respect to the numeral, the speaker encourages the hearer to entertain alternatives. Since the speaker has used a scalar which is associated with information about what is sufficiently similar to the numeral, i.e. what is sufficiently scalarly close to the numeral, the hearer faces pragmatic pressure to use this information in computing alternatives and will therefore end up with a set of alternatives that looks like approximation, unless there is additional relevant information to rule certain alternatives out. We have also seen how a head movement analysis has trouble capturing the correct semantics, and I have suggested that analyzing AI as involving a post-nominal relative structure can provide coherent semantics.

References

Bianchi, V. (1999). *Consequences of Antisymmetry: Headed Relative Clauses*. Mouton de Gruyter.

[8]See Kayne (1994), who proposes that the head of a relative clause is base-generated within the relative clause and moves out. This type of movement may be independently needed in a relative analysis of AI to account for the noun's case, which resembles the phenomenon of Inverse Case Attraction (Bianchi 1999), which also receives a movement analysis.

[9]Billings suggests that in AI two prosodic words exchange position, e.g. [dvadcat']^{PrWd} [knig]^{PrWd} $\rightarrow$ [knig]^{PrWd} [dvadcat']^{PrWd}. So, it could be the case that a single prosodic word containing the noun moves out of the relative structure in (18), but cannot move over anything larger than a single prosodic word.

Billings, L. (1995). *Approximation in Russian and the Single-Word Constraint*. Ph. D. thesis, Princeton University.

Billings, L. and M. Yadroff (1996). *Prosodic Correspondence in Syntax: Russian Approximative Inversion*, pp. 45–71. Foris, Dortrecht.

Cinque, G. (2005). The dual source of adjectives and phrasal movement in the Romance DP. manuscript.

Kayne, R. (1994). *The Antisymmetry of Syntax*. The MIT Press.

Krifka, M. (1995). Common nouns: a contrastive analysis of chinese and english. In G. N. Carlson and F. J. Pelletier (Eds.), *The Generic Book*, pp. 398–411. University of Chicago Press.

Pereltsvaig, A. (2006). Passing by cardinals: In support of head movement in nominals. In *Proceeding of the 14th Annual Workshop on Formal Approaches to Slavic Linguistics*, pp. 277–292.

Wiese, H. (2003). *Numbers, Language, and the Human Mind*. Cambridge: Cambridge University Press.

Yadroff, M. and L. Billings (1998). The syntax of approximative inversion in Russian (and the general architecture of nominal expressions). In *Proceeding of the 6th Annual Workshop on Formal Approaches to Slavic Linguistics*, pp. 319–338.